



New dimensionless correlation-equations for laminar free convection heat transfer in real gases with high wall-fluid temperature differences

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Abstract

Two-dimensional steady free convection heat transfer between isothermal vertical flat plates and real gases, also at thermodynamic states characterized by a compressibility factor significantly different than unity, was studied theoretically by taking into account the actual temperature-dependence of the physical properties of the fluids investigated. The laminar boundary-layer governing equations were solved numerically by a control-volume formulation of the finite-difference method, for significantly wide ranges of variability of the gas pressure, of the wall temperature, and of the free stream temperature. Numerical simulations were executed for air, carbon dioxide, and methane. The dimensionless heat transfer data obtained from computations were correlated by two general equations that, in contrast with other correlation-equations available in the literature, may be assumed as independent of the fluids specifically considered and, even more important, cover almost the whole vapor region.

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1. Introduction

Theoretical studies of free convection heat transfer are often performed through the so-called *constant-property approach*, where all the fluid properties are assumed as constant except for density in the buoyancy force term of the momentum equation. However, the results obtained this way are appreciably good as long as the wall-fluid temperature difference is sufficiently small.

In contrast, in all those cases wherein the wall temperature T_W is noticeably different than the free stream temperature T_∞ , namely the temperature ratio T_W/T_∞ is noticeably different than unity, the *variable-property approach* should be used in order to avoid committing even serious errors in the evaluation of the heat transfer rates. Currently, two methods are commonly used to account for the effects of the variable fluid properties when the heat transfer data obtained by either numerical simulations or experiments are expressed through the usual power-law-type dimensionless correlations: (a) the *reference temperature method*, where

a temperature at which the fluid properties have to be calculated is suitably chosen, e.g., the film temperature, and (b) the *property ratio method*, where all the fluid properties are calculated at one of the temperatures peculiar to the problem (e.g., the wall temperature or the free stream temperature, as in the constant-property approach), and the effects of the variability of each physical property upon the heat transfer rates are accounted for through the ratio between the property calculated at the wall temperature to the same property calculated at the free stream temperature, or at the film temperature as well.

One of the first analytical studies on the variable fluid-property problem for laminar free convection on vertical surfaces was performed by Sparrow and Gregg [1], who showed that the constant-property heat transfer results obtained for gases with $0.5 \leq T_W/T_\infty \leq 3.0$ and expressed by a simple relationship of the form $Nu = A Gr^{1/4}$, may be extended to the variable-property situation once the coefficient of volumetric expansion β is replaced by $1/T_\infty$ and all the other fluid properties are evaluated at the reference temperature $T_r = T_W - 0.38(T_W - T_\infty)$. Minkowycz and Sparrow [2] found that the same approach can be applied also to the case of steam once the coefficient 0.38 in the reference temperature expression is replaced by a coefficient 0.46. Churchill

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Nomenclature

b, C	constants used in Eq. (2)
C_P	specific heat at constant pressure . . . $\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$
g	acceleration due to gravity $\text{m}\cdot\text{s}^{-2}$
h_x	local coefficient of convection $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$
h_L	average coefficient of convection . . . $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$
L	length of the plate m
p	pressure Pa
p_c	critical pressure Pa
p_R	reduced pressure, $= p/p_c$
q_x	local heat transfer rate per unit area . . . $\text{W}\cdot\text{m}^{-2}$
T	absolute temperature K
T_f	film temperature, $= (T_W + T_\infty)/2$ K
T_{\max}	maximum temperature K
T_{\min}	minimum temperature K
T_r	reference temperature K
T_{sat}	saturation temperature K
T_W	wall temperature K
T_∞	free stream temperature K
u	longitudinal velocity component $\text{m}\cdot\text{s}^{-1}$
v	normal velocity component $\text{m}\cdot\text{s}^{-1}$
x	longitudinal coordinate m
y	normal coordinate m
Nu	Nusselt number, $= hL/\lambda$
Pr	Prandtl number, $= C_P\mu/\lambda$

Gr	Grashof number, $= g\rho \rho_W - \rho_\infty L^3/\mu^2$
Z	compressibility factor

Greek symbols

β	coefficient of volumetric expansion K^{-1}
λ	thermal conductivity $\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$
μ	absolute viscosity $\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$
θ	absolute wall–fluid temperature difference, $= T_W - T_\infty $ K
ρ	mass density $\text{kg}\cdot\text{m}^{-3}$
ρ_c	critical mass density $\text{kg}\cdot\text{m}^{-3}$
ρ_R	reduced mass density, $= \rho/\rho_c$
ω	normal coordinate m
ξ	longitudinal coordinate m

Subscripts

f	at the film temperature
M	at the M th grid node in the ω -direction
\max	maximum value, maximum value within the boundary layer
\min	minimum value
r	at the reference temperature
W	at the wall temperature
∞	at the free stream temperature

and Chu [3] developed correlation-equations of the form $Nu = f(Pr)Ra^\alpha$ which provide a good representation of a wide variety of experimental data for both liquids and gases once the fluid properties are evaluated at the film temperature $T_f = (T_W + T_\infty)/2$. Brown [4] studied the effects of variability of β , whose dependence on temperature was assumed as linear, which infers a corresponding exponential dependence of the density ρ on temperature, and proposed an equation to estimate the ratio of the Nusselt number for the case of variable β and ρ to the Nusselt number for the case of constant β and ρ . Gray and Giorgini [5] analyzed the validity of the Boussinesq approximation for liquids and gases under the assumption that the fluid properties are linear functions of both temperature and pressure. Clausing and Kempka [6] conducted experiments with gaseous nitrogen at atmospheric pressure, free stream temperatures in the range $80 \text{ K} < T_\infty < 320 \text{ K}$ (thus substantially in the dilute gas state) and temperature ratios T_W/T_∞ in the range between 1 and 2.6 (which then means wall–fluid temperature differences up to 780 K), and concluded that, for the laminar regime, the effects of the variable properties might be well approximated by evaluating the fluid properties at the film temperature. Ghajar and Parker [7] calculated the heat transfer rates for supercritical refrigerant R-114, water and carbon dioxide and, for each of them, proposed a suitable reference temperature. Sabhapathy and Cheng [8] studied the effects of the variability of both the viscosity μ

and the coefficient of volumetric expansion β upon the stability of the laminar boundary-layer flow of liquids, showing that the temperature-dependent $\mu = \mu(T)$ stabilizes the flow along a heated wall and destabilizes it along a cooled wall, whilst the temperature-dependent $\beta = \beta(T)$ initially stabilizes the flow for a heated wall but further downstream it destabilizes the flow. Shang and Wang [9,10] performed theoretical studies of the variable fluid-property problem by assuming the ideal gas behaviour and fluid properties variable with the absolute temperature according to a simple power law of the form $\gamma/\gamma_\infty = (T/T_\infty)^{n(\gamma)}$ where γ denotes the generic fluid property. In detail, they proposed a set of dimensionless local heat transfer correlation-equations, each of them specialized for a particular fluid, put in the form $Nu_{x,\infty} = A[n(\gamma)]f(Pr)(Gr_{x,\infty})^{1/4}(T_W/T_\infty)^{B[n(\gamma)]}$. More recently, Fontana et al. [11] and Cianfrini et al. [12] studied the variability of the physical properties of gases where a fast chemical reaction of dissociation-recombination takes place and calculated their effects upon heat transfer from a vertical flat plate with both conditions of isothermal and isoflux boundary surface, respectively.

However, despite the relatively large amount of work done in this field, it seems worthwhile noticing that the majority of the studies carried out up to now concern ideal gases or anyway fluids sufficiently far removed from their critical conditions and hence enough stable in terms of fluid-property changes. Indeed, several science and engineering

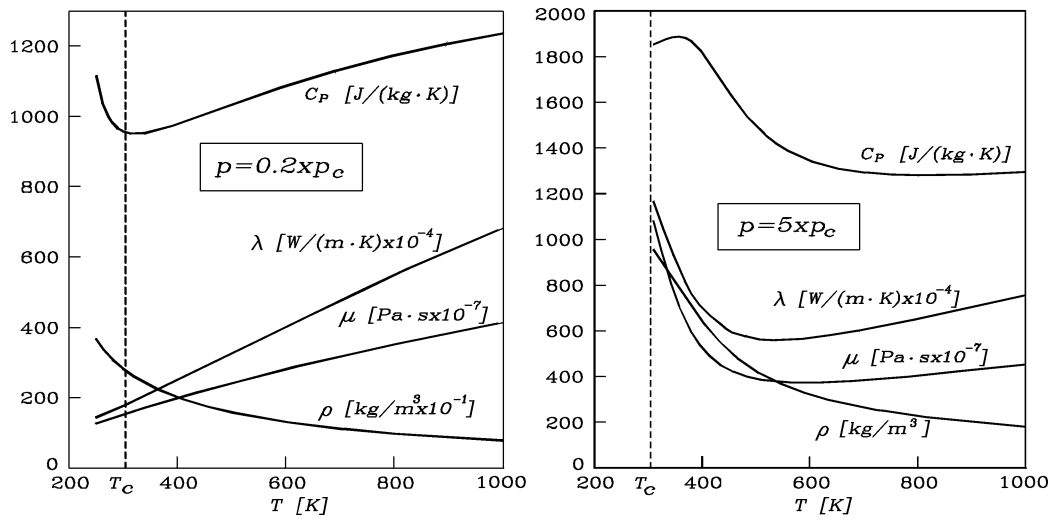


Fig. 1. Distributions of the physical properties of CO₂ vs. absolute temperature for $p = 0.2p_c$ and for $p = 5p_c$.

applications that involve the use of gases may occur not only at high wall–fluid temperature differences, but also at very high pressures or near saturation, i.e., at thermodynamic states where the ideal gas model definitely fails and the temperature-dependence of the fluid properties cannot be described by simple monotonic laws. This is, e.g., shown for carbon dioxide, whose distributions of density, specific heat at constant pressure, viscosity, and thermal conductivity versus the absolute temperature are depicted in Fig. 1 for two different pressures, $p = 0.2p_c$ and $p = 5p_c$ respectively, where p_c denotes the critical pressure.

Therefore, since very little attention has been paid to this problem, either in the past or even more recently, a numerical study of laminar free convection heat transfer between isothermal vertical flat plates and real gases is performed for significantly wide ranges of variability of the gas pressure, of the wall temperature, and of the free stream temperature, with the basic aim of deriving dimensionless correlations that, in contrast with other equations available in the literature, were not fluid-specialized and, above all, covered most of the thermodynamic states comprised in the vapour region.

2. Theoretical analysis

Two-dimensional steady free convection heat transfer between an isothermal vertical flat plate and a real gas was studied theoretically with the following basic assumptions: (a) laminar boundary-layer flow; (b) negligible viscous dissipation, negligible work against the gravity field, and negligible thermal diffusion; (c) uniform free stream temperature of the fluid; and (d) uniform pressure throughout the whole boundary layer.

The analysis was carried out for dry air, carbon dioxide, and methane, whose equation of state was assumed to be $p/\rho = Z(p_R, \rho_R)T$, where the compressibility factor Z is

expressed as a function of both the reduced pressure p_R and the reduced density ρ_R . The values of the fluid properties and of the compressibility factor with varying temperature and pressure were derived from the extended experimental data-collection by Vargaftik et al. [13]. In particular, where the property data were presented in the form of tables rather than through equations, cubic interpolations of the tabular data with respect to both temperature and pressure were executed.

3. Governing equations

The laminar boundary-layer governing equations are expressed in the (ξ, ω) coordinate system, as also shown in Fig. 2:

$$\xi = x, \quad \omega = \frac{y}{\Psi} \quad (1)$$

being

$$\Psi = \Psi(\xi) = C\xi^b \quad (2)$$

where C and b are constants.

Continuity equation

$$\frac{\partial \rho u}{\partial \xi} + \frac{1}{\Psi} \frac{\partial \rho v}{\partial \omega} - \frac{\omega}{\Psi} \frac{d\Psi}{dx} \frac{\partial \rho u}{\partial \omega} = 0 \quad (3)$$

Momentum equation

$$\rho u \frac{\partial u}{\partial \xi} + \left(\frac{\rho v}{\Psi} - \rho u \frac{\omega}{\Psi} \frac{d\Psi}{dx} \right) \frac{\partial u}{\partial \omega} - \frac{1}{\Psi^2} \frac{\partial}{\partial \omega} \left(\mu \frac{\partial u}{\partial \omega} \right) + g|\rho - \rho_\infty| = 0 \quad (4)$$

which, taking into account Eq. (2), becomes:

$$\frac{\partial}{\partial \xi} (\rho u \Psi u) + \frac{\partial}{\partial \omega} \left[u \left(\rho v - \rho u \omega \frac{d\Psi}{dx} \right) - \frac{\mu}{\Psi} \frac{\partial u}{\partial \omega} \right] + g|\rho - \rho_\infty| = 0 \quad (5)$$

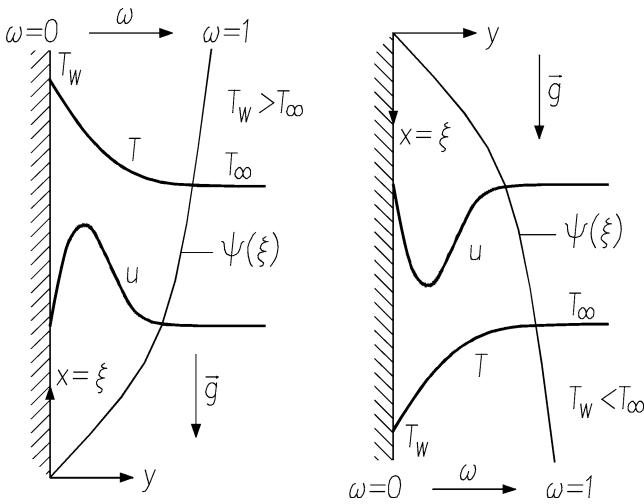


Fig. 2. Physical model and coordinate system.

Energy equation

$$\frac{\partial}{\partial \xi} (\rho u \Psi T) + \frac{\partial}{\partial \omega} \left[T \left(\rho v - \rho u \omega \frac{d\Psi}{dx} \right) \right] - \frac{1}{C_p} \frac{\partial}{\partial \omega} \left(\lambda \frac{\partial T}{\partial \omega} \right) = 0 \quad (6)$$

The boundary conditions are derived by imposing a uniform temperature T_W and zero velocity at the plate surface, as well as zero velocity, zero velocity gradient, uniform temperature T_∞ and zero temperature gradient at a great distance from the plate. Hence:

$$\begin{aligned} \omega = 0, \xi \geq 0: \quad u = v = 0, \quad T = T_W \\ \omega \rightarrow \infty, \xi \geq 0: \quad u = v = 0, \quad \frac{\partial u}{\partial \omega} = \frac{\partial v}{\partial \omega} = 0 \\ T = T_\infty, \quad \frac{\partial T}{\partial \omega} = 0 \end{aligned} \quad (7)$$

4. Numerical method of solution

The set of governing equations (3), (4) and (6) with the boundary conditions (7) was solved numerically by a control-volume formulation of the finite-difference method by assuming the coordinate ξ as a one-way coordinate. In the derivation of the discretized equations, the downstream values of the dependent variables along the ξ -direction were assumed to prevail over the entire $\Delta\xi$ of the control volume (fully implicit scheme), whilst the exponential scheme was used along the ω -direction [14].

The interval-spacings $\Delta\xi$'s were assumed less than or equal to $\Psi(\xi)/4$. The values of both the coefficient C and the exponent b in the expression of $\Psi(\xi)$ given in Eq. (2) were calculated by a trial-and-error procedure so that the thermal and velocity boundary layers were largely contained within the range $0 \leq \omega \leq 1$, as clarified below. In particular, the values of C and b corresponding to the case of a fluid with constant properties evaluated at the film temperature were assumed as first tentative values.

As concerns the discretization scheme along the ω -direction, the number M of grid nodes was chosen so that at least 50 grid nodes were contained within the thinnest of the two boundary layers, and uniform interval-spacings $\Delta\omega$ and $2\Delta\omega$ were assumed in the range $0 \leq \omega \leq 0.7$ and in the range $0.7 < \omega \leq 1$, respectively.

For each spatial interval in the ξ -direction, the calculations along the ω -direction were executed by imposing the following conditions at $\omega = 1$:

$$T_M = T_\infty \quad \text{and} \quad u_M = 0 \quad (8)$$

where subscript M refers to the grid node of the ω -direction discretization that corresponds to $\omega = 1$, i.e., the last grid node along the ω -direction. Indeed, owing to the strong nonlinearities consequent to the temperature-dependence of the fluid properties, within each discretization step along the ξ -direction successive iterations along the ω -direction were required for the solution of both the temperature field and the velocity field up to convergence attainment. In particular, the solution was considered to be fully converged when the absolute value of the mass source and the percent changes of the dependent variables at any grid node from iteration to iteration were smaller than prescribed values, i.e., 10^{-4} and 10^{-5} , respectively.

For each integration step along the ξ -direction, once the iterative process along the ω -direction was terminated, the “flatness” of the temperature and velocity profiles at $\omega = 1$ was verified through the following conditions:

$$\begin{aligned} |T_{M-3} - T_\infty| \leq |T_W - T_\infty| \times 10^{-4} \quad \text{and} \\ |u_{M-3}| \leq u_{\max} \times 10^{-4} \end{aligned} \quad (9)$$

where subscript max refers to the maximum value within the velocity boundary layer.

In all those cases Eqs. (9) were not satisfied, the whole computational procedure described above was repeated by assuming new further-approximation values of the constants C and b in Eq. (2), until appropriate solutions, i.e., solutions responding to Eqs. (9), were obtained. In more details, successive approximations values $C^{(n)} = 2 \times C^{(n-1)}$ and $b^{(n)} = 1.1 \times b^{(n-1)}$ were assumed, where superscripts $(n-1)$ and (n) denote the $(n-1)$ th and the n th computational procedure, respectively.

5. Numerical experiments and heat transfer analysis

Numerical simulations were performed for: (a) the three gases cited above, i.e., dry air, carbon dioxide, and methane; (b) different values of the length L of the vertical plate in the range 0.01 to 0.5 m; and (c) different values of the gas pressure p , of the wall temperature T_W , and of the free stream temperature T_∞ , as indicated in detail in Table 1. Specifically, almost 6000 numerical experiments were performed, nearly 2000 for each of the gases examined. For each case analyzed, once both the temperature field and

Table 1
Pressure and temperature ranges of the numerical simulations

Gas	p_c [MPa]	T_c [K]	p_{min} [MPa]	p_{max} [MPa]	T_{min} [K]	T_{max} [K]	θ_{min} [K]	θ_{max} [K]		
Air	132.5	$\cong 3.8$	0.02	50	$p \leq 0.02$ $T_{min} = 72$	$0.02 < p < 0.85 p_c$ $T_{min} = T_{sat} + 5$	$p \geq 0.85 p_c$ $T_{min} = 140$	1700	10	1620
CO ₂	304.2	7.38	0.3	100	$p \leq 0.6$ $T_{min} = 230$	$0.6 < p < 0.85 p_c$ $T_{min} = T_{sat} + 5$	$p \geq 0.85 p_c$ $T_{min} = 320$	1000	10	760
CH ₄	190.8	4.64	0.1	100	$p \leq 0.12$ $T_{min} = 132$	$0.12 < p < 0.85 p_c$ $T_{min} = T_{sat} + 5$	$p \geq 0.85 p_c$ $T_{min} = 200$	1000	10	860

the velocity field were obtained, the local heat transfer rate per unit area from the surface of the plate to the gas was calculated by the Fourier’s law:

$$q_x = -\lambda_W \left. \frac{\partial T}{\partial \omega} \right|_{\omega=0} \quad (10)$$

where λ_W is the thermal conductivity of the fluid evaluated at the wall temperature, $\lambda_W = \lambda(T_W)$, and the temperature gradient at the wall was evaluated by assuming a second-order temperature profile among each wall node and the next two fluid nodes along the ω -direction.

The local values and the average value of the coefficient of convection, denoted with h_x and h_L , respectively, were then derived:

$$h_x = \frac{q_x}{T_W - T_\infty} \quad (11)$$

$$h_L = \frac{1}{L} \int_0^L h_x dx \quad (12)$$

where the integral was approximated by the trapezoid rule.

6. Heat transfer correlation-equations

Leaving aside what we could name the fluid-specialized approach followed by some authors who aimed either at defining a reference temperature or at developing heat transfer dimensionless equations which were specific for each fluid investigated (see, e.g., Ghajar and Parker [7] and Shang and Wang [9,10], respectively), the general reference temperature and property ratio methods cited in the introduction section are the methods more usually used to account for the effects of the variable fluid properties, even due to the power and simplicity of employment of the arising correlations.

In this framework, also for the sake of a generality as larger as possible, in the present study we decided to try to correlate the heat transfer numerical data by following one of the two methods of above, or a combination of them, by using correlation-equations of the simple power-law type.

The best results in terms of standard deviation of data and maximum percent error were obtained: (a) by evaluating the fluid thermophysical properties either at the wall temperature T_W , for the calculation of the average Nusselt number

Nu_W , or at the free stream temperature T_∞ , for the calculation of both the Grashof number Gr_∞ and the Prandtl number Pr_∞ ; and (b) by including the density ratio (ρ_W/ρ_∞) and the viscosity ratios (μ_W/μ_∞) and (μ_f/μ_∞) among the dimensionless parameters, where subscripts W, ∞ and f indicate that the fluid property is evaluated at the wall temperature, at the free stream temperature, and at the film temperature, respectively.

In addition, as in the twin papers by Shang and Wang [9, 10], the best data-fit was achieved by considering two different correlations, according as the flow occurs along a heated or a cooled plate. This result could actually be expected by considering that the way the fluid properties change is strongly dependent on whether the temperature of the fluid increases or decreases, which may imply either stabilization or destabilization of the boundary-layer flow, as also observed by Sabhapathy and Cheng for liquids [8].

The two correlation-equations arisen from the logarithmic multiple-regression of the numerical heat transfer data are as follows:

$$Nu_W = 0.523 Gr_\infty^{0.25} Pr_\infty^{0.26} \left(\frac{\mu_W}{\mu_\infty} \right)^{-0.57} \left(\frac{\rho_W}{\rho_\infty} \right)^{0.26} \quad (T_W > T_\infty) \quad (13)$$

with

$$0.84 < Gr_\infty < 9.9 \times 10^{12}, \quad 0.69 < Pr_\infty < 1.45$$

$$0.49 < \mu_W/\mu_\infty < 4, \quad 0.14 < \rho_W/\rho_\infty < 0.99$$

$$T_\infty/T_c < 0.95 \quad \text{or} \quad T_\infty/T_c > 1.1$$

$$T_\infty/T_{sat} \geq 1.05$$

$$Nu_W = 0.568 Gr_\infty^{0.25} Pr_\infty^{0.59} \left(\frac{\mu_W}{\mu_\infty} \right)^{-0.49} \left(\frac{\mu_f}{\mu_\infty} \right)^{-0.87} \quad (T_W < T_\infty) \quad (14)$$

with

$$0.06 < Gr_\infty < 7.8 \times 10^{12}, \quad 0.69 < Pr_\infty < 1.24$$

$$0.37 < \mu_W/\mu_\infty < 2.04, \quad 0.73 < \mu_f/\mu_\infty < 1.23$$

$$T_\infty/T_c < 0.8 \quad \text{or} \quad T_\infty/T_c > 1.25, \quad T_\infty/T_{sat} \geq 1.05$$

The standard deviations of data of Eqs. (13) and (14) are 0.040 and 0.049, respectively, whilst the maximum absolute

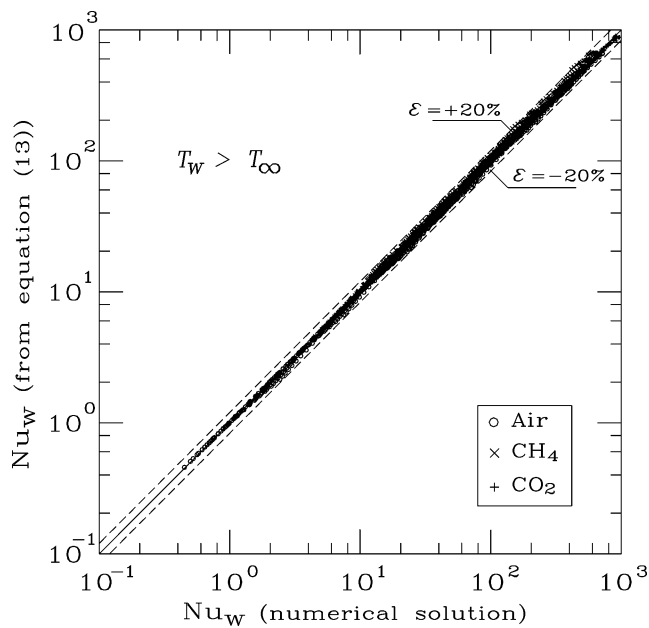


Fig. 3. Comparison between the average Nusselt numbers predicted by Eq. (13) and those obtained through the numerical simulations.

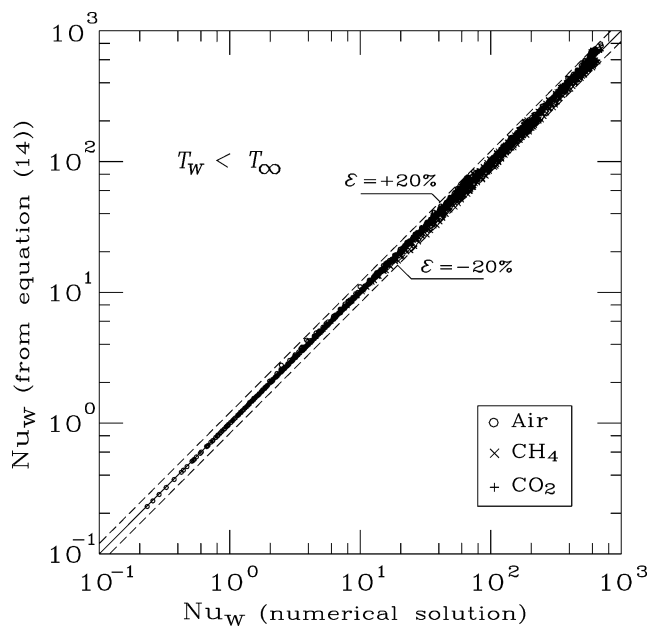


Fig. 4. Comparison between the average Nusselt numbers predicted by Eq. (14) and those obtained through the numerical simulations.

values of the relative error are 0.19 and 0.21, respectively, as shown in Figs. 3 and 4.

In addition, since in both Figs. 3 and 4 the heat transfer data obtained for the three fluids analyzed are completely “scrambled” in the range of the average Nusselt number between 10^{-1} and 10^3 across which they are scattered, this may lead to the conclusion that the validity of Eqs. (13) and (14) might reliably be extended to gases different from those investigated here.

7. Comparison with other heat transfer correlation-equations

Eqs. (13) and (14) comprise many more thermodynamic states than those comprised by many correlation-equations available in the literature, as those obtained through numerical studies wherein the assumption of ideal gas behaviour is made. This is, e.g., the case of the correlation proposed by Sparrow and Gregg for the evaluation of the local and of the overall Nusselt number [1], as well as the case of the sets of fluid-specialized double equations introduced by Shang and Wang for the evaluation of the local Nusselt number [9, 10], whose use outside their range of validity may be also significantly unsound. In this regard, the order of the errors which may be committed by using these correlations across the entire range of validity of Eqs. (13) and (14), i.e., across practically the whole vapor region, is shown in Fig. 5, where the performance of the correlation obtained by Churchill and Chu for laminar flow [3] is also evaluated, both for $T_W > T_\infty$ and for $T_W < T_\infty$ (see Appendix A for details on equations). Here, in order to account for the different definitions of the Grashof number and of the reference temperature, the coefficient of convection is used instead of the Nusselt number so as making possible to compare the results of the correlations cited above among them and with the results of the numerical simulations performed.

In particular, it may be observed that, in many cases, either negative or positive non-negligible errors may be committed in the evaluation of the coefficient of convection, according as the fluid flow occurs along a heated or a cooled plate, thus stressing the necessity to study these two cases separately.

8. Conclusions

Two-dimensional steady free convection heat transfer between an isothermal vertical flat plate and a real gas was studied numerically for wide ranges of variability of the gas pressure, of the wall temperature, and of the free stream temperature, so as to investigate a number of thermodynamic states of the vapor region as largest as possible. Basic aim of the paper was the development of one or more heat transfer dimensionless correlations that (a) were characterized by ranges of validity definitely wider than those of the equations available in the literature, and (b) might reliably be assumed as independent of the fluid specifically considered, i.e., so that their coefficients and exponents were fluid-independent.

The scope of the paper has been achieved by correlating the dimensionless heat transfer data through two general correlation-equations (one for the case of heat transferred from the wall to the fluid, the other for the opposite case) which are characterized by large generality, by relatively small standard deviations of data, and by ranges of validity that span across practically the whole vapour region.

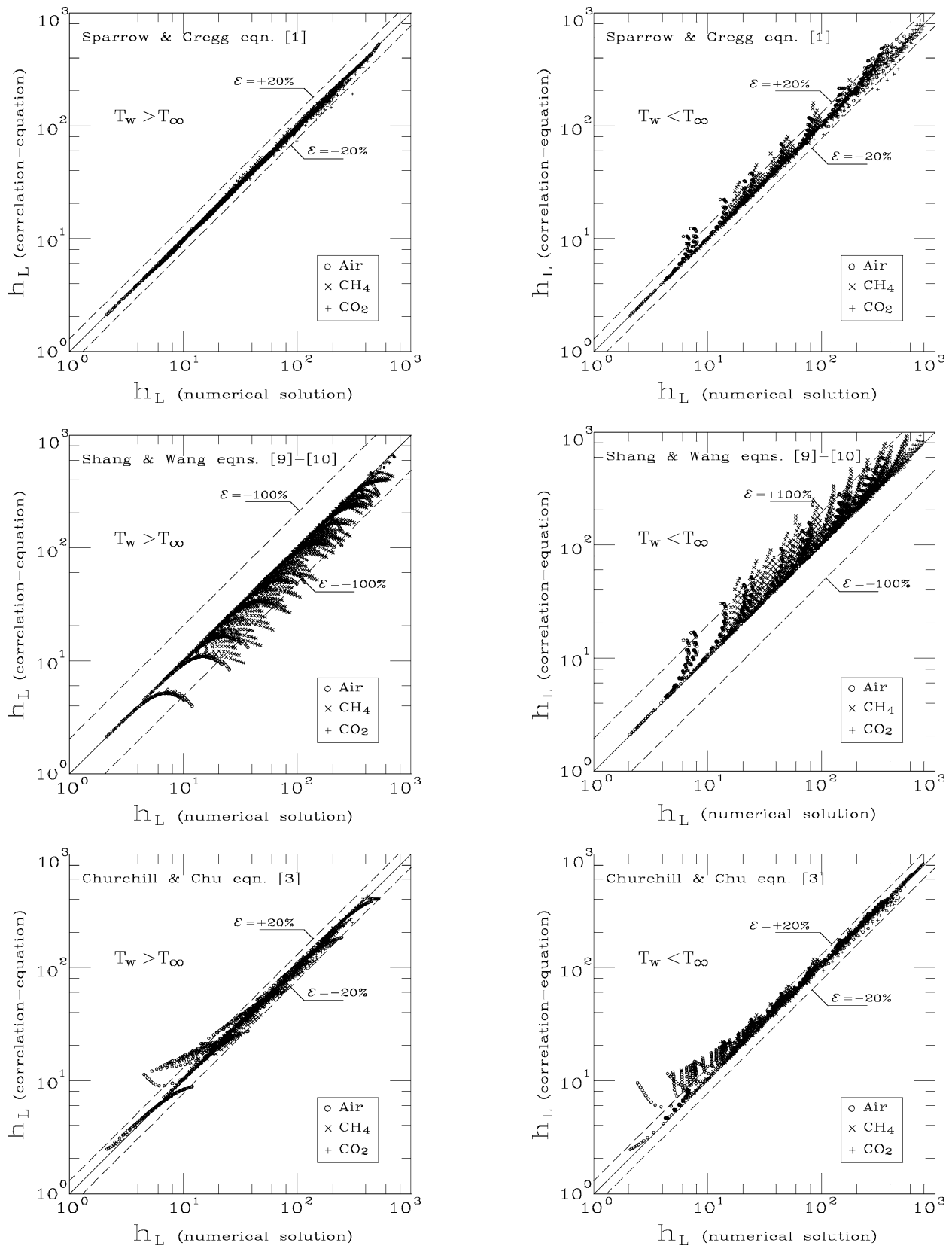


Fig. 5. Comparison between the coefficients of convection predicted by Sparrow and Gregg [1], by Shang and Wang [9,10], by Churchill and Chu [3] and those obtained through the numerical simulations carried out for both cases of $T_w > T_\infty$ and $T_w < T_\infty$.

Appendix A

- Correlation by Sparrow and Gregg [1]

$$\frac{Nu_x}{Gr_x^{1/4}} = 0.353 = \frac{Nu}{(4/3)Gr^{1/4}}$$

with fluid properties evaluated at the reference temperature:

$$T_r = T_W - 0.38(T_W - T_\infty)$$

- Correlation by Churchill and Chu [3]

$$Nu = 0.68 + \frac{0.670Ra^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}}$$

with fluid properties evaluated at the reference temperature:

$$T_r = \frac{T_W + T_\infty}{2}$$

- Correlations by Shang and Wang
Monoatomic and biatomic gases [9]

$$Nu_{x,\infty} = \frac{\psi(Pr)}{\sqrt{2}} Gr_{x,\infty}^{1/4} \left(\frac{T_W}{T_\infty} \right)^{n(\lambda)-m}$$

where:

$$\psi(Pr) = 0.567 + 0.186 \ln(Pr)$$

$$m = 0.35n(\lambda) + 0.29n(\mu) + 0.36 \quad \text{for } T_W > T_\infty$$

$$m = 0.42n(\lambda) + 0.34n(\mu) + 0.24 \quad \text{for } T_W < T_\infty$$

for air: $n(\lambda) = 0.81$ and $n(\mu) = 0.68$.

Polyatomic gases [10]

$$Nu_{x,\infty} = [1 + 0.3n(C_P)] \frac{\psi(Pr)}{\sqrt{2}} Gr_{x,\infty}^{1/4} \left(\frac{T_W}{T_\infty} \right)^{n(\lambda)-m}$$

where:

$$\psi(Pr) = 0.567 + 0.186 \ln(Pr)$$

$$m = 0.35n(\lambda) + 0.29n(\mu) + 0.36 \quad \text{for } T_W > T_\infty$$

$$m = 0.42n(\lambda) + 0.34n(\mu) + 0.24 \quad \text{for } T_W < T_\infty$$

for CO₂: $n(\lambda) = 1.3$, $n(\mu) = 0.88$, $n(C_P) = 0.34$; for CH₄:

$n(\lambda) = 1.29$, $n(\mu) = 0.78$, $n(C_P) = 0.534$.

References

- [1] E.M. Sparrow, J.L. Gregg, The variable fluid-property problem in free convection, *Trans. ASME* 80 (1958) 879–886.
- [2] W.J. Minkowycz, E.M. Sparrow, Free convection heat transfer to steam under variable property conditions, *Internat. J. Heat Mass Transfer* 9 (1966) 1145–1147.
- [3] S.W. Churchill, H.H.S. Chu, Correlation equations for laminar and turbulent free convection from a vertical plate, *Internat. J. Heat Mass Transfer* 18 (1975) 1323–1329.
- [4] A. Brown, The effect on laminar free convection heat transfer of the temperature dependence of the coefficient of volumetric expansion, *J. Heat Transfer* 97 (1975) 133–135.
- [5] D.D. Gray, A. Giorgini, The validity of the Boussinesq approximation for liquids and gases, *Internat. J. Heat Mass Transfer* 19 (1976) 545–551.
- [6] A.M. Clausing, S.N. Kempka, The influence of property variations on natural convection from vertical surfaces, *J. Heat Transfer* 103 (1981) 609–612.
- [7] A.J. Ghajar, J.D. Parker, Reference temperature for supercritical laminar free convection on a vertical flat plate, *J. Heat Transfer* 103 (1981) 613–616.
- [8] P. Sabhapathy, K.C. Cheng, The effects of temperature-dependent viscosity and coefficient of thermal expansion on the stability of laminar natural convective flow along an isothermal vertical surface, *Internat. J. Heat Mass Transfer* 29 (1986) 1521–1529.
- [9] D.-Y. Shang, B.-X. Wang, Effect of variable thermophysical properties on laminar free convection of gas, *Internat. J. Heat Mass Transfer* 33 (1990) 1387–1395.
- [10] D.-Y. Shang, B.-X. Wang, Effect of variable thermophysical properties on laminar free convection of polyatomic gas, *Internat. J. Heat Mass Transfer* 34 (1991) 749–755.
- [11] D.M. Fontana, M. Corcione, A. Lo Monaco, L. Santarpia, Laminar free convection from a vertical plate in partly dissociated gases, *Internat. J. Heat Mass Transfer* 43 (2000) 1113–1120.
- [12] C. Cianfrini, M. Corcione, D.M. Fontana, Laminar free convection from a vertical plate with uniform surface heat flux in chemically reacting systems, *Internat. J. Heat Mass Transfer* 45 (2002) 319–329.
- [13] N.B. Vargaftik, Y.K. Vinogradov, V.S. Yargin, *Handbook of Physical Properties of Liquids and Gases: Pure Substances and Mixtures*, 3rd Edition, Begell House, New York, 1996.
- [14] D.B. Spalding, A novel finite difference formulation for differential expressions involving both first and second derivatives, *Internat. J. Numer. Methods Engrg.* 4 (1972) 551.